Level A

When you add the two numbers you get a certain answer. When you subtract the same two numbers you get the same answer. In this problem we will call that a perfect pair.

Can you find two numbers that are a perfect pair?

If you think it is impossible explain why.

If you have found a perfect pair, explain why they are a perfect pair.

If it is possible, are there more than one perfect pair? How many are there?

Explain all you know about perfect pairs and what is special about them.
Level B

Each shoe represents a digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). The style of shoe always represents the same digit in all of the number sentences. Examine the number sentences.

Pair each shoe with a digit. Explain your reasoning.
Level C

A perfect trio involves three whole numbers. Using the three numbers, add the first two numbers together then divide the sum by the third number. Using the same three numbers, subtract the second from the first number and then multiple the difference by the third. The trio is perfect if the two outcomes are equal.

Can you find three whole numbers that are perfect trios?

If not, how can you show that there are not any perfect trios? Is there a way to proof there are no perfect trios? Explain completely.

If you can find a perfect trio, is there more than one perfect trio? If so, how many? List the trio(s) you found. How do you know if you found them all? Describe any special characteristics of perfect trios. How can you go about finding them? Explain completely.
Level D

In this problem a perfect pair is defined as two numbers whose sum is equal to its product.

Explore these perfect pairs.

If you cannot find any perfect pairs prove that a perfect pair cannot exist.

If you find perfect pairs, then generalize your findings and describe the relationship of the number pairs. Illustrate the set of perfect pairs using multiple representations (words, symbols, graphs, tables, diagrams).
Level E

In this problem a perfect pair is defined as two numbers whose sum is equal to its quotient.

Explore these perfect pairs.

If you cannot find any perfect pairs prove that a perfect pair cannot exist.

If you find perfect pairs, then generalize your findings and describe the relationship of the number pairs. Illustrate the set of perfect pairs using multiple representations (words, symbols, graphs, tables, diagrams).
Primary Version Level A

Materials: Number cards 0 – 9, a paper bag, counters, paper and pencil.

Discussion on the rug: Teacher says to the class, “I have numbers in my bag.” The teacher shows the class the content of the bag and shows the cards with single digit, counting them from 0 to 5. The teacher asks a student to pick out a number from the bag and show it to the class. The teacher says, “What is the name of that number. Show on your fingers how many it is.” The class responds. The teacher then asks a second person to pick a different number from the bag and shows it to the class. The teacher says, “If we added the two numbers, how many would we have?” The teacher asks for answers and then asks students to prove it using counters, or drawings, or demonstrate with fingers, etc. (The process can be repeated until the students have comfort with the procedure of adding on. The teacher now demonstrates an activity that requires students to find the missing addend. First the teacher picks a number out of the bag and shows the class. The teacher pulls a 1 card from the bag but turns it face down. The teacher mentally calculates and writes the sum on the board and states, “What was the second number I would have had to pick to get this answer. The teacher asks the students to respond and justify their answer).

Back at their desks: Each student has access to a bag with numbers 0 – 9, counters, pencil and paper. The teacher picks a number from the bag. The teacher picks a second number, hiding it and writes the sum on the board. The students are given time to calculate the missing addend using a process that makes sense to them. This game is repeated several times including a time when zero is used as the first number drawn and also as the second number drawn. At the end of the investigation, have students either discuss or dictate a response to the prompt: “How did you find the hidden number? Tell me how to find it no matter which numbers are picked.”
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